# Predictive navigation of an autonomous vehicle with nonholonomic and minimum turning radius constraints ${ }^{\dagger}$ 

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#### Abstract

A key feature of an autonomous vehicle is the ability to re-plan its motion from a starting configuration (position and orientation) to a goal configuration while avoiding obstacles. Moreover, it should react robustly to uncertainties throughout its maneuvers. We present a predictive approach for autonomous navigation that incorporates the shortest path, obstacle avoidance, and uncertainties in sensors and actuators. A car-like robot is considered as the autonomous vehicle with nonholonomic and minimum turning radius constraints. The results (arcs and line segments) from a short-est-path planner are used as a reference to find action sequence candidates. The vehicle's states and their corresponding probability distributions are predicted to determine a future reward value for each action sequence candidate. Finally, an optimal action policy is calculated by maximizing an objective function. Through simulations, the proposed method demonstrates the capability of avoiding obstacles as well as of approaching a goal. The regenerated path will incorporate uncertainty information.


Keywords: Autonomous vehicle; Motion planning; Nonholonomic constraint; Obstacle avoidance; Uncertainties

## 1. Introduction

The development of intelligent autonomous vehicles is being intensively reported in many areas due to their wide applications such as service robots, material handling vehicles, military operations, etc. It is desirable that an autonomous vehicle has such capabilities to develop its motion planning that can guide itself from an initial to a goal configurations, while avoiding obstacles to prevent collision, and also overcoming sensors' and actuators' uncertainties. This paper presents the development of a navigation method to cover the aforementioned requirements.

Autonomous vehicles and mobile robots are nonholonomic systems. It is known that a nonholonomic

[^0]system cannot be stabilized by any time-invariant and smooth feedback control law [1]. Several methods have been proposed to address the stabilization problem of nonholonomic systems and they can be classified as problems of open loop control $[2,3]$ and timevarying or discontinuous feedback control [4, 5]. The open loop strategies are generally able to guide the system from an initial to goal configurations, but the robustness is not guaranteed. While a closed loop strategy potentially provides more robust results, the dynamics of the closed loop system may show oscillatory and chattering response in the way to reach the goal configuration [6]. In this paper, the open loop strategy is implemented as a control method. To guarantee its robustness, the iterative design presented in [7] is adopted.

The obstacle avoidance problem adds one level higher difficulty in autonomous vehicle navigation. Early works in obstacle avoidance include the poten-
tial field method [8], vector field histogram (VFH) [9], etc. However, those approaches do not consider the nonholonomic constraint of the vehicle. The methods that take into account nonholonomic constraints are the curvature velocity method (CVM) [10], dynamic window approach (DWA) [11], VFH + [12], lane based method [13], etc. However, such methods do not consider the desired orientation at the goal point. They just assume that the autonomous robots/vehicles can pivot (rotate in place) to meet the desired orientation. Considering that many autonomous vehicles in industries are subjected to a minimum turning radius constraint due to the steering angle limitations of the vehicle wheels, it is necessary to develop an algorithm that considers the orientation of the vehicle at the goal position.

Robustness of an autonomous vehicle control is also required to deal with uncertainties that might appear either from sensors and actuators or system models. Due to the presence of uncertainties, the vehicle should reconsider a selected action sequence regarding the uncertainty information. In [14-16], uncertainties in robotics applications are viewed from a probabilistic perspective. This leads to the Markov decision process (MDP) and its extension, partially observable Markov decision process (POMDP). The POMDP explicitly models the sensors' and actuators' uncertainties and estimates the states of the vehicle. The POMDP method is also known as a predictive approach since it predicts a reward value that a vehicle will obtain if an action sequence is executed. The prediction involves the probability distributions of the states, instead of only the states, since the true states are not completely known. In [17], the POMDP method provides a direct action to a mobile robot and it covers planning and obstacle avoidance ability. The mobile robot architecture for reflexive avoidance of moving obstacles is also proposed in [18].

We propose a predictive navigation approach considering sensors' and actuators' uncertainty information. The uncertainties are modeled as probability distributions over vehicle's states and control inputs. The states and their probability distributions are predicted through the extended Kalman filter (EKF) algorithm [19] to obtain a predicted area that might be occupied by a vehicle with respect to a particular input. This predicted area is used to determine a reward value that is calculated from a reward value function. The collision event is set to be the first priority condition of the reward value function [20]. For
other conditions, the reward value is determined based on the difference configuration between the predicted states and the goal configuration. An optimal action sequence is obtained by maximizing an objective function that is the sum of the reward values of multiple time step prediction. The action sequence candidates are obtained by modifying the resulting action sequence from the shortest-path planner for a car-like system. By applying such action candidates, the nonholonomic and minimum turning radius constraints of the vehicle are explicitly considered.

The proposed navigation can be implemented to the robot software which its architecture has object oriented characteristics for changing data among functions [21]. The path following control is assumed to be available such as presented in [22, 23]. Also, real time sensors that are used to reconstruct the robot and obstacle state are assumed to be available such as proposed in [24-26].

From our investigation, there are no published papers yet on an autonomous vehicle navigation control which includes the uncertainties and, at the same time, considers the nonholonomic and minimum turning radius constraints of the vehicle. The contributions of this paper are as follows. First, the proposed method links the initial and goal configurations, and performs the obstacle avoidance ability for autonomous vehicle by taking into account the nonholonomic and minimum turning radius constraints. The path is generated by considering the desired orientation at the goal point. Second, the proposed method gives a capability to maintain its decision on planning. Based on uncertainty information, the autonomous vehicle can decide whether it can pass through a narrow space or it should find a wider space while approaching the goal. During its operation, an autonomous vehicle sometimes is required to approach an object. A particular example of such an object is a pallet to be engaged by an autonomous forklift. The proposed method is also able to supervise two objectives: avoiding obstacles and approaching an object.

The paper is structured as follows. In Section 2, a shortest-path planner is discussed. Section 3 describes a predictive navigation method using a reward value function. Simulation results of the proposed navigation method and its comparison with other obstacle avoidance method are presented in Section 4. The conclusions are given in Section 5.

## 2. Shortest path planner for a car-like robot

The vehicle model considered here is a car-like robot. It describes the kinematics constraints of a wheeled vehicle moving with a minimal turning radius. The configuration of the vehicle $(x, y, \theta)$ is described by $x$ and $y$ positions and the orientation $\theta$ with respect to the $x$-axis of the global coordinate.

The kinematic model of a car-like robot with minimal turning radius is formulated as follows.

$$
\begin{align*}
& {\left[\begin{array}{c}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{array}\right]=\left[\begin{array}{c}
\cos \theta \\
\sin \theta \\
0
\end{array}\right] v+\left[\begin{array}{c}
0 \\
0 \\
1 / R
\end{array}\right] \omega,}  \tag{1}\\
& |v| \leq v_{\max }, \text { and }|\omega| \leq \omega_{\max }
\end{align*}
$$

where $v$ and $\omega$ are the linear and rotational velocities, respectively. $R$ is the minimum turning radius that the vehicle can achieve.

The shortest-path planning problem is described by a car-like robot moving forward and backward. It has been proven in [2] that the shortest-path for such systems consists of straight line segments and arcs of circles. In other words, the shortest path between two configurations for a car-like robot is developed by a pair sequence of linear and rotational velocities in the domain of $v \in\left\{-v_{\max }, v_{\text {max }}\right\}$ and $\omega \in\left\{-\omega_{\max }, 0\right.$, $\left.\omega_{\max }\right\}$.
The optimal path is found in a sufficient family of paths that can be divided into nine groups as listed in Table 1, where the notation $C$ denotes an arc of a circle, $S$ denotes a straight line segment, the separator | denotes a changing forward-backward directions, and the subscripted parameters determine the time-length of the corresponding path segment.

Table 1. Families of the shortest path for a car-like robot [2].

| Notation | Range of parameter |
| :---: | :---: |
| $C_{\alpha}\left\|C_{\beta}\right\| C_{\varepsilon}$ | $0 \leq \alpha \leq \pi, \quad 0 \leq \beta \leq \pi, \quad 0 \leq \varepsilon \leq \pi$ |
| $C_{\alpha} \mid C_{\beta} C_{\varepsilon}$ | $0 \leq \alpha \leq \beta, \quad 0 \leq \varepsilon \leq \beta, \quad 0 \leq \beta \leq \pi / 2$ |
| $C_{\alpha} C_{\beta} \mid C_{\varepsilon}$ | $0 \leq \alpha \leq \beta, \quad 0 \leq \varepsilon \leq \beta, \quad 0 \leq \beta \leq \pi / 2$ |
| $C_{\alpha} C_{\beta} \mid C_{\beta} C_{\varepsilon}$ | $0 \leq \alpha<\beta, \quad 0 \leq \varepsilon<\beta, \quad 0 \leq \beta \leq \pi / 2$ |
| $C_{\alpha}\left\|C_{\beta} C_{\beta}\right\| C_{\varepsilon}$ | $0 \leq \alpha<\beta, \quad 0 \leq \varepsilon<\beta, \quad 0 \leq \beta \leq \pi / 2$ |
| $C_{\alpha} \mid C_{\pi / 2} S_{\gamma} C_{\pi / 2} C_{\beta}$ | $0 \leq \alpha<\pi / 2, \quad 0 \leq \beta<\pi / 2, \quad 0 \leq \gamma$ |
| $C_{\alpha} \mid C_{\pi / 2} S_{\gamma} C_{\beta}$ | $0 \leq \alpha<\pi / 2, \quad 0 \leq \beta<\pi / 2, \quad 0 \leq \gamma$ |
| $C_{\beta} S_{\gamma} C_{\pi / 2} \mid C_{\alpha}$ | $0 \leq \alpha \leq \pi, \quad 0 \leq \beta \leq \pi / 2, \quad 0 \leq \gamma$ |
| $C_{\alpha} S_{\gamma} C_{\beta}$ | $0 \leq \alpha \leq \pi / 2, \quad 0 \leq \beta \leq \pi / 2, \quad 0 \leq \gamma$ |

The action sequence candidates are found as follows. For given initial and goal configurations, one can determine the defining time-length parameter for each path listed in Table 1, calculate their time-length of arcs and line segments, and take the minimum result as the shortest path. The shortest path will be used as a reference action sequence. The action sequence candidates are determined by modifying the parameters (time-length parameter and the direction of arc and line segment) of the reference action sequence.

## 3. Predictive navigation approach

In this section, we introduce the procedure of the proposed predictive navigation approach. The belief state $\mathbf{b}_{k}$ is defined as a probability distribution over a set of states of the autonomous vehicle. An EKF algorithm is used to obtain belief state prediction. In determining an optimal action sequence, the update step of the EKF algorithm is not processed yet.

The discrete time model of the system in Eq. (1) disturbed by the errors of the linear and rotational velocities is given by

$$
\begin{equation*}
\mathbf{x}(k+1)=\mathbf{A}(\mathbf{x}(k))\left(\mathbf{u}(k)+\mathbf{w}_{u}(k)\right) \tag{2}
\end{equation*}
$$

where

$$
\left.\begin{array}{l}
\mathbf{x}(k)=[x(k) y(k) \theta(k)
\end{array}\right]^{T}, \begin{array}{ccc}
\mathbf{A}(\mathbf{x}(k))=\left[\begin{array}{ccc}
\cos \theta(k) & \sin \theta(k) & 0 \\
0 & 0 & 1
\end{array}\right]^{T} \\
\mathbf{u}(k)=\left[\begin{array}{ll}
v(k) & \omega(k)
\end{array}\right]^{T}, \text { and } \\
\mathbf{w}_{u}(k)=\left[\begin{array}{ll}
w_{v}(k) & w_{\omega}(k)
\end{array}\right]^{T}
\end{array}
$$

The errors of the linear and rotational velocities are described by $w_{v}(k)$ and $w_{\omega}(k)$, respectively, which are assumed to be white with covariance matrix $\mathbf{Q}(k)$.

The state prediction based on the EKF is

$$
\begin{align*}
\hat{\mathbf{x}}^{-}(k+1)= & \mathbf{A}(\hat{\mathbf{x}}(k)) \mathbf{u}(k),  \tag{3}\\
\mathbf{P}^{-}(k+1)= & \mathbf{F}(k) \mathbf{P}(k) \mathbf{F}(k)^{T} \\
& +\mathbf{G}(k) \mathbf{Q}(k) \mathbf{G}(k)^{T} \tag{4}
\end{align*}
$$

where $\hat{\mathbf{x}}(k)$ and $\mathbf{P}(k)$ denote the state vector and error covariance matrix estimated at time $k$, respectively. The predicted state and covariance matrix at time $k+1$ are described by $\hat{\mathbf{x}}^{-}(k+1)$ and $\mathbf{P}^{-}(k+1)$, respectively. $\mathbf{F}(k)$ and $\mathbf{G}(k)$ are the Jacobians of $\mathbf{A}$ with respect to $\mathbf{x}(k)$ and $\mathbf{w}_{u}(k)$ evaluated at $k$,
respectively, which are obtained as follows.

$$
\begin{align*}
& \mathbf{F}(k)=\left[\begin{array}{ccc}
1 & 0 & -v(k) T \sin \theta(k) \\
0 & 1 & v(k) T \cos \theta(k) \\
0 & 0 & 1
\end{array}\right],  \tag{5}\\
& \mathbf{G}(k)=\left[\begin{array}{cc}
-T \cos \theta(k) & 0 \\
-T \sin \theta(k) & 0 \\
0 & -T
\end{array}\right], \tag{6}
\end{align*}
$$

where $T$ is the sampling time.
For $N$-step prediction, Eqs. (3), (4) are computed $N$-times. At each step prediction, a predicted ellipse area is produced by the calculation of the variance errors of the vehicle's position in $x$ - and $y$-coordinates. This area presents a possible location of the autonomous vehicle.

By using the predicted area, a reward value function $\psi_{k}\left(\mathbf{b}_{k}, \mathbf{u}_{k}\right)$ with respect to the goal and obstacle positions is introduced as

$$
\begin{aligned}
& \psi_{k}\left(\mathbf{b}_{k}, \mathbf{u}_{k}\right)= \\
& \left\{\begin{array}{cc}
1 & \text { if }\left\|\boldsymbol{\Gamma}\left(\hat{\mathbf{x}}_{j}-\mathbf{x}_{\text {goal }}\right)\right\| \leq \rho \\
-1 & \text { if } \frac{\left(\hat{x}_{j}-x_{o_{i}}\right)^{2}}{\sigma_{x x}^{2}}+\frac{\left(\hat{y}_{j}-y_{o_{i}}\right)^{2}}{\sigma_{y y}^{2}} \leq 1, \text { (7) } \\
1-\frac{\left\|\Gamma\left(\hat{\mathbf{x}}_{j}-\mathbf{x}_{\text {goal }}\right)\right\|}{\left\|\Gamma\left(\hat{\mathbf{x}}_{k}-\mathbf{x}_{\text {goal }}\right)\right\|} & \text { others }
\end{array}\right.
\end{aligned}
$$

where $\rho$ is the small acceptable value of the weighted norm $\left\|\Gamma\left(\hat{\mathbf{x}}_{j}-\mathbf{x}_{\text {goal }}\right)\right\|$ between the predicted state $\hat{\mathbf{x}}_{j}$ at $j=k+1, . ., N$ and the goal state $\mathbf{x}_{\text {goal }}, \Gamma$ is a diagonal weighting matrix. The values of $\rho$ and $\Gamma$ are determined as follows. First, the acceptable values of position errors $p_{e}=\left[\begin{array}{ll}x_{e} & y_{e}\end{array}\right]^{T}$ and orientation error $\theta_{e}$ at the goal point are selected. Second, a weighting matrix $\Gamma$ is adjusted so that the orientation error has a proportional value with the position errors. Finally, the value of $\rho$ is deter-
 $x$ - and $y$-positions are described by $\hat{x}_{j}$ and $\hat{y}_{j}$, respectively. The variances of the predicted position in the $x$ - and $y$-coordinates are denoted as $\sigma_{x x}^{2}$ and $\sigma_{y y}^{2}$, respectively, which are the two first diagonals of the predicted covariance $\mathbf{P}^{-}(k+1)$ obtained from Eq. (4). The position of $i$-th obstacle in the global coordinate is presented by $x_{o_{i}}$ and $y_{o_{i}}$.

In the implementation of Eq. (7), the second reward
value is placed at the first priority condition; hence, the action sequence that projects an area occupied by obstacles will always be rewarded with the value of -1 . The functions that guarantee the vehicle to meet the goal configuration are the first and the third reward value functions. The first reward value function gives a reward value +1 for any part of action sequence that achieves the acceptable configuration at the goal point. For other conditions, the action sequence that produces a smaller value of weighted norm $\| \Gamma$.( $\left.\hat{\mathbf{x}}_{j}-\mathbf{x}_{\text {goal }}\right) \|$ between the predicted state $\hat{\mathbf{x}}_{j}$ and the goal state $\mathbf{x}_{\text {goal }}$ obtains a larger reward value. Using the reward value function in Eq. (7), the action sequence candidates are examined through an objective function to determine the optimal action sequence for a multi-time steps.

The objective function is given by

$$
\begin{equation*}
V_{k}^{*}\left(\mathbf{b}_{k}, \mathbf{u}_{k}\right)=\max _{u \in U}\left(\sum_{j=k}^{N} \gamma^{j-k} \psi_{j}\left(\mathbf{b}_{j}, \mathbf{u}_{j}\right)\right), \tag{8}
\end{equation*}
$$

where $\gamma \in(0,1]$ is a discount factor that determines the importance of the future rewards that will be received by the vehicle.

Once an optimal action sequence of multi-time steps is obtained, then a one time-step action sequence will be executed and the states measurement will be performed. The recent belief state is obtained through the update step of EKF.

The observation model at time $k+1$ is modeled as follows.

$$
\begin{equation*}
\hat{\mathbf{z}}(k+1)=\mathbf{H}(\hat{\mathbf{x}}(k+1))+\mathbf{v}_{z}, \tag{9}
\end{equation*}
$$

where $\mathbf{H}$ is the observation function and $\mathbf{v}_{z}$ is the measurement noise, which is assumed to be white with covariance matrix $\boldsymbol{\Phi}$. The belief state update after a measurement $\mathbf{z}(k+1)$ is calculated by the following procedure.

$$
\begin{align*}
\hat{\mathbf{x}}(k+1)= & \hat{\mathbf{x}}^{-}(k+1)+\mathbf{K}(k+1)(\mathbf{z}(k+1) \\
& \left.-\mathbf{H}(k+1) \hat{\mathbf{x}}^{-}(k+1)\right),  \tag{10}\\
\mathbf{P}(k+1)= & \mathbf{P}^{-}(k+1) \\
& -\mathbf{K}(k+1) \mathbf{S}(k+1) \mathbf{K}(k+1)^{T},  \tag{11}\\
\mathbf{K}(k+1)= & \mathbf{P}^{-}(k+1) \mathbf{J}(k+1)^{T} \mathbf{S}(k+1)^{-1},  \tag{12}\\
\mathbf{S}(k+1)= & \mathbf{J}(k+1) \mathbf{P}^{-}(k+1) \mathbf{J}(k+1)^{T} \\
& +\boldsymbol{\Phi}(k+1), \tag{13}
\end{align*}
$$

where $\mathbf{K}(k+1)$ is the filter gain that reduces the effect of noise on the state estimates, $\mathbf{S}(k+1)$ is the
measurement prediction covariance, and $\mathbf{J}(k+1)$ is the Jacobian of $\mathbf{H}(k+1)$ with respect to $\hat{\mathbf{x}}(k+1)$.

## 4. Simulation results

### 4.1 Obstacle avoidance

The obstacle avoidance performance between the proposed method and VFH + are compared in Fig. 1. The initial and goal configurations are points $\mathrm{A}(-15$, $39.5,3.14)$ and $B(-20,34,3.14)$, respectively. The obstacles C are described by points in the $x$ - and $y$ coordinates. The end configuration of the executed path is shown by a vehicle with dark color.

Fig. 1(a) shows that the proposed method is able to guide the autonomous vehicle from one configuration

Table 2. Comparison of the final configurations between the proposed method and the $\mathrm{VFH}+$ method.

| The desired final con- <br> figuration | $x$ <br> $(-20.00 \mathrm{~m})$ | $y$ <br> $(34.00 \mathrm{~m})$ | $\theta$ <br> $(3.14 \mathrm{rad})$ |
| :---: | :---: | :---: | :---: |
| The proposed method | -19.79 | 33.97 | 3.22 |
| The VFH + | -19.95 | 34.01 | 3.75 |



Fig. 1. Comparison of the proposed method and the VFH + : Obstacle avoidance.
to another while also avoiding obstacles, considering the desired orientation at the goal point. In Fig. 1(b), although the final position can be achieved, the resulting final orientation in VFH + method is largely different from the desired orientation. The comparison of the final configurations between the proposed method and VFH + method is given in Table 2.

### 4.2 Decision making based on uncertainty information

In this simulation, we examine the consideration of the proposed method to the uncertainty information. The initial configuration is set to $(7,13,0)$, the goal configuration is set to ( $12,15, \pi / 2$ ), and some obstacle points are placed as shown in Fig. 2.

In Fig. 2(a), the standard deviations of the position and orientation measurement errors are defined as $\sigma_{x x}=\sigma_{y y}=0.1$ and $\sigma_{\theta \theta}=0.1 \pi$, respectively. The standard deviations for linear and rotational velocities are $\sigma_{v v}=0.1$ and $\sigma_{\omega \omega}=0.1$, respectively. It shows that the executed path from the proposed method

(a) Small variance errors

(b) The regenerated path: Large variance errors

Fig. 2. The regenerated path when the variance errors of the states are large.
guides the autonomous vehicle to pass through a narrow passage.

In Fig. 2(b), the standard deviations of the position and orientation measurement errors are set to $\sigma_{x x}=\sigma_{y y}=0.2$ and $\sigma_{\theta \theta}=0.2$, respectively. The standard deviations for the linear and rotational velocities errors are $\sigma_{v v}=0.2$ and $\sigma_{\omega \omega}=0.2$, respectively. At first, the action policy of the proposed method follows the shortest-path. At some distance, it finds that the projected belief state from the shortestpath is occupied by obstacles. At this point, the action policy of the proposed method decides a wider passage to pass. The resulting path of the vehicle depends on the setting of the initial belief state of the position and orientation of the vehicle, and also the determination of sensors and actuators uncertainties.
The left pointing arrow means that the vehicle moves backward $(v=-1)$. This motion is created from the iterative design which generates a new shortest-path from the present state as a new initial point to the goal configuration. With these results, we conclude that the resulting path of the proposed method relies on the predicted variance errors which are predicted from the uncertainties information.

### 4.3 Approaching object problem

In this simulation scenario, we check the feasibility of the proposed method when the vehicle approaches an object. The scenario is related to the situation in the working area where the autonomous vehicle usually has to approach an object. In this situation, the autonomous vehicle should determine what action should be executed, avoid obstacles or approach the goal. The initial, via point, and goal configurations are set to $(6,13,0),(12,14, \pi / 2)$, and $(12.5,16.5,0)$, respectively. The target is an object located right in front of the goal point.

The reward value function in Eq. (7) maintains the resulting path by predicting reward values from the initial vehicle's configuration. It manages the motion of the vehicle to approach the object while also avoiding obstacles. Fig. 3 demonstrates the capability of an autonomous vehicle in dealing with the aforementioned problems.

## 5. Conclusions

A predictive navigation approach for an autonomous vehicle considering uncertainties has been pre sented. The method utilizes a belief state prediction to


Fig. 3. The performance of obstacle avoidance of the proposed method while approaching an object.
include the uncertainty information, which is the sensors' and actuators' errors. The optimal action sequence is obtained by optimizing an objective function that is a function of future rewards given to the autonomous vehicle.

The predictive approach is confirmed with the shortest-path planner for a car-like robot model to include the nonholonomic and minimum turning radius constraints. The proposed method could satisfy the desired orientation at the goal point. By using a reward value function, the vehicle has a capability to cope the problem of approaching object problem while also avoiding obstacles. Moreover, the method can also be implemented as a unified model that provides localization, planning, and also obstacle avoidance. Through simulations, it was confirmed that the resulting path of the proposed method was based upon the uncertainty information.

Future work involves the application of the prediction navigation approach in real time, integrating the high level planner, also combining a prediction method of moving obstacles to perform efficient obstacle avoidance in dynamic environment. A research for the autonomous vehicle to determine its own belief state is also needed. Furthermore, the application of N -steps prediction belief state to multi vehicles and cooperation will be examined.

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## References

[1] I. Kolmanovsky and N. H. McClamroch, Developments in nonholonomic control problems, IEEE Control System Magazine, 15 (6) (1995) 20-36.
[2] P. Souéres and J. P. Laumond, Shortest paths synthesis for a car-like robot, IEEE Transactions on Automatic Control, 41 (5) (1996) 672-688.
[3] R. M. Murray and S. S. Sastry, Nonholonomic motion planning: Steering using sinusoids, IEEE Transactions on Automatic Control, 38 (5) (1993) 700-716.
[4] C. Samson, Control of chained systems application to path following and time varying point-stabilization of mobile robots, IEEE Transactions on Automatic Control, 40 (1) (1995) 64-77.
[5] E. Valtolina and A. Astolfi, Local robust regulation of chained systems, System Control Letters, 29 (3) (2003) 231-238.
[6] B. M. Kim and P. Tsiotras, Controllers for unicycletype wheeled robots: Theoretical results and experimental validation, IEEE Transactions on Robotics and Automation, 18 (3) (2002) 294-307.
[7] P. Lucibello and G. Oriolo, Robust stabilization via iterative state steering with an application to chained form systems, Automatica, 37 (1) (2001) 71-79.
[8] O. Khatib, Real-time obstacle avoidance for manipulators and mobile robots, International Journal of Robotics Research, 5 (1) (1986) 90-98.
[9] J. Borenstein and Y. Koren, The vector field histo-gram-fast obstacle avoidance for mobile robots, IEEE Transactions on Automatic Control, 7 (3) (1991) 278-288.
[10] R. Simmons, The curvature-velocity method for local obstacle avoidance, Proc. of the IEEE Int. Conf. Robotics and Automation, Minneapolis, MN, USA, (1996) 3375-3382.
[11] D. Fox, W. Burgard, and S. Thrun, The dynamic window approach to collision avoidance, IEEE Robotics and Automation Magazine, 4 (1) (1997) 2333.
[12] I. Ulrich and J. Borenstein, VFH+: Reliable obstacle avoidance for fast mobile robots, Proc. of the IEEE Int. Conf. Robotics and Automation, Leuven, Belgium, (1998) 1572-1577.
[13] N. Y. Ko, R. Simmons and K. S. Kim, A lane based avoidance method for mobile robot navigation, KSME International Journal, 17 (11) (2003) 1693-1731.
[14] S. Thrun, Probabilistic algorithms in robotics, $A I$ Magazine, 21 (4) (2000) 93-109.
[15] D. H. Choi, S. J. Lee and H. H. Yoo, Dynamic analysis of multi-body systems considering probabilistic properties, Journal of Mechanical Science and Technology, 19 (1) (2005) 113-139.
[16] Y. S. Kim and K. S. Hong, A tracking algorithm for autonomous navigation of AGVs in an automated container terminal, Journal of Mechanical Science and Technology, 19 (1) (2005) 72-86.
[17] A. Foka and P. Trahanias, Real time hierarchical POMDPs for autonomous robot navigation, Robotics and Autonomous System, 55 (7) (2007) 561-571.
[18] K. S. Hong, T. A. Tamba and J. B. Song, Mobile robot architecture for reflexive avoidance in moving obstacles, Advanced Robotics, 22 (13-14) (2008) 1397-1420.
[19] Y. Bar-Shalom, X. R. Li and T. Kirubarajan, Estimation with applications to tracking and navigation, Wiley-Interscience, New York, USA, (2001).
[20] N. Y. Ko, D. J. Seo and R. G. Simmons, Collisionfree motion coordination of heterogeneous robots, Journal of Mechanical Science and Technology, 22 (11) (2008) 2090-2098.
[21] C. S. Kim, K. S. Hong and Y.-S. Han, PC-based off-line programming in the shipbuilding industry: Open architecture, Advanced Robotics, 19 (4) (2005) 435-458.
[22] Y. Gao, C. G. Lee and K. T. Chong, Receding horizon tracking control for mobile robots with time delay, Journal of Mechanical Science and Technology, 22 (12) (2008) 2403-2416.
[23] T. A. Tamba, H. Hong and K. S. Hong, A path following control of an unmanned autonomous forklift, International Journal of Control, Automation, and Systems, 7 (1) (2009) 113-122.
[24] J. Bae, S. Lee and J.-B. Song, Use of coded infrared light for mobile robot localization, Journal of Mechanical Science and Technology, 22 (7) (2008) 1279-1286.
[25] S.-J. Lee, J.-H. Lim and D.-W. Cho, Feature-map building using sparse sonar data in a home-like environment, Journal of Mechanical Science and Technology, 21 (1) (2007) 74-82.
[26] Y. S. Kim, K. S. Hong and S. K. Sul, Anti-sway control of container cranes: Inclinometer, observer, and state feedback, vol. 2, no. 3, International Journal of Control, Automation, and Systems, 2 (4) (2004) 435-449.


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